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LETTER TO THE EDITOR

Broken rotation symmetry in the fractional quantum Hall system

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Abstract. We demonstrate that the two-dimensional electron system in a strong perpendicular magnetic field has stable states which break rotational but *not* translational symmetry. The Laughlin fluid becomes unstable to these states in quantum wells whose thickness exceeds a critical value which depends on the electron density. The order parameter at 1/3 reduced density resembles that of a nematic liquid crystal, in that a residual twofold rotation axis is present in the low-symmetry phase. At filling factors 1/5 and 1/7, there are states with fourfold and sixfold axes, as well. We discuss the experimental detection of these phases.

The construction of the Laughlin fluid and its identification with the fractional quantum Hall effect (FQHE) marked the discovery of a qualitatively new many-body phase [1]. Until that discovery, it was generally assumed that the only phases present in the two-dimensional electron system were the usual liquid phase and the Wigner solid, both of which are present in the classical one-component Coulomb plasma phase diagram. A hexatic phase is also a possibility in this system [2] as well as in the logarithmic potential case perhaps more relevant to the FQHE [3]. This theoretical background, and the experimental discovery of phases which seem to have properties unlike either the Laughlin liquid or the Wigner solid [4] ('Hall insulator'), led us to an investigation of the possibility of broken rotational symmetry (BRS) in the two-dimensional electron system in strong field. We found that several liquid crystal-like phases occur, though these appear to resemble more closely nematic than hexatic phases.

We begin with the case where the electron density n satisfies $n = \frac{1}{3}/2\pi l^2$, where l is the magnetic length: $l^{-2} = eB/\hbar c$ and B is the external field, taken to be in the negative z -direction. In this paper we work in the limit of large field. This is the density for the 1/3 quantum Hall state. Consider the following wavefunction for the disc-shaped system:

$$\Psi_{\alpha}(z_i) = \prod_{i < j}^N [(z_i - z_j)(z_i - z_j - \alpha l)(z_i - z_j + \alpha l)] \exp\left(-\sum_i |z_i|^2/4l^2\right). \quad (1)$$

Here $z_i = x_i + iy_i$ and i and j are particle indices. α is a complex number. This wavefunction is antisymmetric in the particle indices for all α , lies entirely in the lowest Landau level, and reduces to the Laughlin wavefunction at $\alpha = 0$. It also shares with the Laughlin

wavefunction the characteristic of having uniform density (far from the edges of the disc). However, the two-particle correlation function is a different matter:

$$g_\alpha(\mathbf{r}) = \frac{N(N-1)}{n^2} \left(\prod_{i>2}^N \int d^2z_i \right) |\Psi_\alpha|^2 / \left(\prod_i \int d^2z_i \right) |\Psi_\alpha|^2. \quad (2)$$

In this equation $\mathbf{r} = (x_1 - x_2, y_1 - y_2)$, and N is the total number of electrons. Translational symmetry is not broken for the small values of α with which we are concerned, as we shall demonstrate below. Thus g_α is a function only of the difference variable \mathbf{r} . In contrast to the Laughlin liquid, however, it does depend on the direction of \mathbf{r} . Let us take α to be real. Then the equivalent classical plasma interaction corresponding to Ψ_α is a logarithmic interaction between rodlike charged objects lying along the x -axis. Accordingly, Ψ_α represents a BRS state [5]. In general $(\text{Re } \alpha, \text{Im } \alpha)$ is a director, not a vector, order parameter, since $\Psi_\alpha = \Psi_{-\alpha}$. Thus $g_\alpha(\mathbf{r})$ does not have full rotation symmetry for $\alpha \neq 0$ but does satisfy $g_\alpha(x, y) = g_\alpha(-x, y) = g_\alpha(x, -y) = g_\alpha(-x, -y)$.

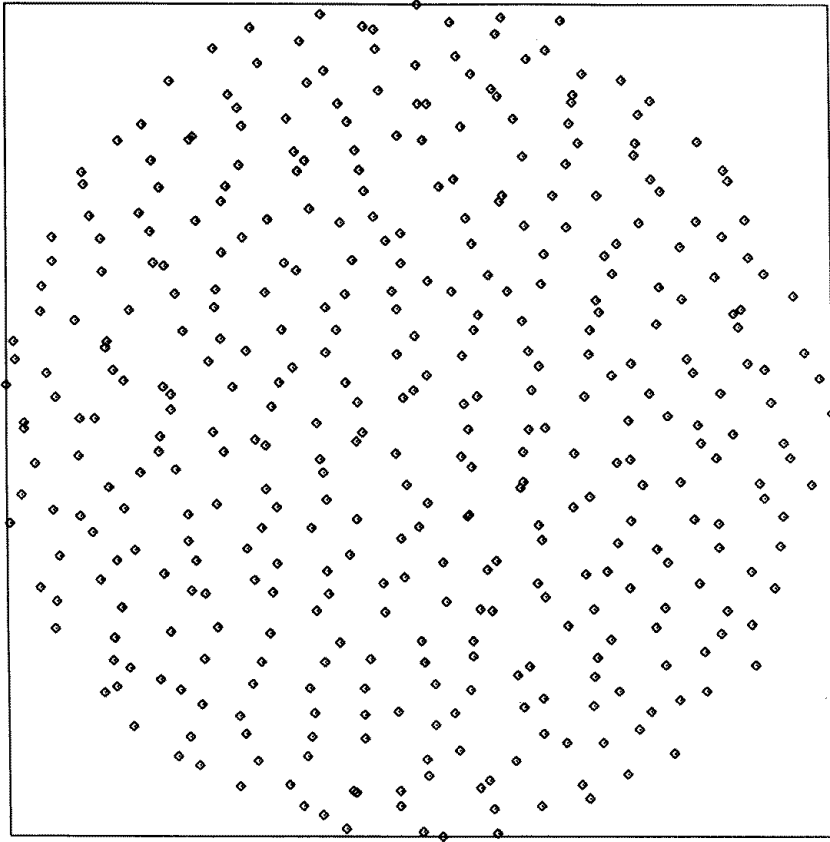


Figure 1. A typical configuration of the particles in the Monte Carlo simulation of the state given by (1) with $\alpha = 3.2$. The anisotropy of the correlations is clearly evident.

To give a physical picture of this state, we display a typical configuration in a Monte Carlo simulation governed by the probability distribution $|\Psi_\alpha|^2$ in figure 1, with $\alpha = 3.2$.

One notes immediately the stripes along the director, reminiscent of a nematic state. This value of α is unphysically large and is chosen for purposes of illustration only.

Under what conditions is such a BRS state stable? Or, given an interelectron potential $V(\mathbf{r}_i - \mathbf{r}_j)$, when do we have $U_\alpha = \langle \Psi_\alpha | V | \Psi_\alpha \rangle < U_0 = \langle \Psi_0 | V | \Psi_0 \rangle$? It is easy to see, expanding the polynomial in (1) and taking the limit $|\mathbf{r}| \rightarrow 0$ in (2), that $g_\alpha(\mathbf{r}) \sim r^2$ rather than $g_\alpha(\mathbf{r}) \sim r^6$. Thus for very short-range V , the Laughlin state is always favoured, as is well known [6, 7].

In order to compare the $\alpha = 0$ with the $\alpha \neq 0$ state we performed Monte Carlo simulations of the equivalent classical plasmas with 200 particles in the disc geometry, and compared the energies of candidate ground states. (We believe that improvement of the wavefunction by, for example, quantum Monte Carlo techniques [8] will not affect energy differences very much.) In order to minimize the finite-size effect we compute the energies of only 25 particles closest to the centre of the disc. We have checked that this procedure gives the accepted value for the energy per particle in the case of a pure Coulomb potential and $\alpha = 0$.

From the computations we found that the Laughlin state is favoured for a pure Coulomb potential $V(r) = e^2/\epsilon r$. The actual potential between electrons in a real two-dimensional layer of finite thickness is softer, owing to the averaging of the charge density over the third dimension. This has been discussed in detail by numerous authors and the choice of potential depends on the shape of the well. We shall take the simple form of Zhang and Das Sarma [9]. They showed that $V(r) = e^2/\epsilon(r^2 + \lambda^2)^{1/2}$ is a reasonable approximation for a square quantum well and that $\lambda \approx 0.2t$, where t is the layer thickness of the well. To understand the effect that this modification of the potential will have, note that the total energy is given by

$$U_\alpha = \frac{ne^2}{2\epsilon} \int \frac{d^2r}{(r^2 + \lambda^2)^{1/2}} [g_\alpha(\mathbf{r}) - 1]. \quad (3)$$

We now plot the angle averaged correlation functions for different values of α in figure 2. There is incipient solid order as α increases, in that there is much more tendency towards being able to identify shells of neighbours. At short distances, the correlation function is proportional to r^2 , just as that for the Wigner crystal is, and in contrast to the r^6 behaviour of the Laughlin state. Overall, the softer potential favours finite α : the correlation hole has a larger effective radius, even though it is not as 'deep'. The difference between the energy per particle for a BRS state with $\alpha = 1$ and the Laughlin state for different values of λ is plotted in figure 3. There is a critical value λ_c at which the system undergoes a transition to finite α . We compute this to be $\lambda_c = 4.1 \pm 1.5$, which corresponds to a thickness of $t = 1600 \text{ \AA}$, when $B = 10 T$. This transition is second order, unlike the transition to the Wigner solid, which is probably first order. It has recently been pointed out that changing t can induce the liquid–solid transition by a mechanism similar to that proposed here [10]. The critical value of t for the $m = 1/3$ density is similar to that computed here, suggesting that the energy balance between Laughlin, BRS, and crystalline states is a subtle one. We expect that the BRS state occupies a fairly narrow range of parameter space between the liquid and the crystal states, by analogy with hexatic phases. It is clear, in any case, that this range of thickness values is experimentally accessible [11]. The energy balance between the BRS and Wigner crystal state is currently under investigation [12].

The correlations are oscillatory even to infinite distances in a true crystalline state. Inspection of figure 2 shows that the correlation function for the BRS state is flat at large distances, demonstrating that long-range translational symmetry is not broken, and justifying the identification of these states as liquid crystal states.

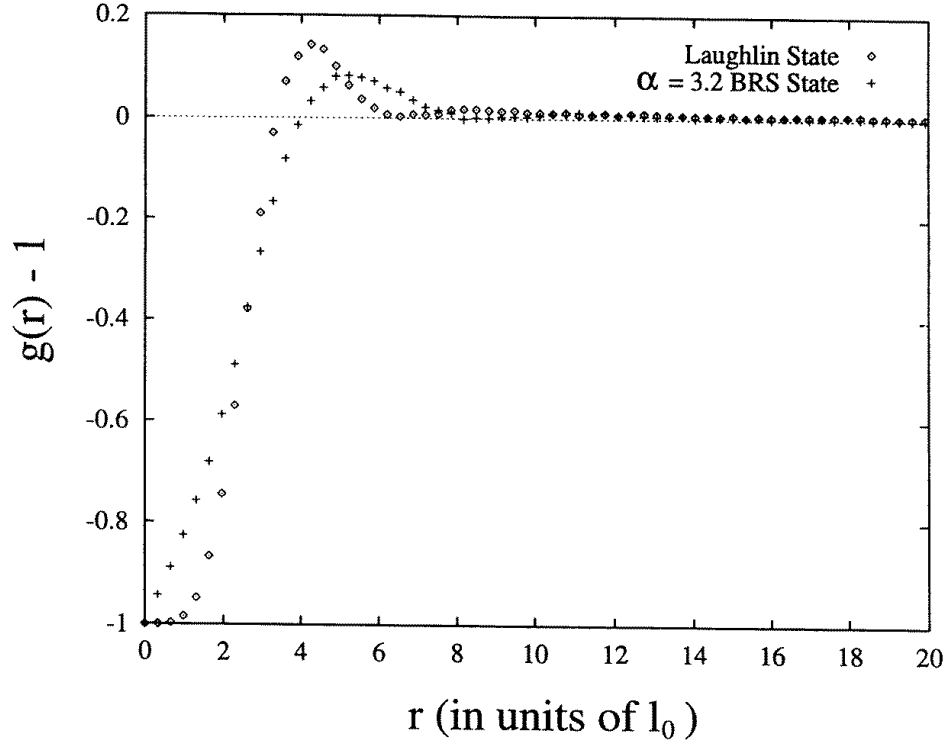


Figure 2. The angle averaged pair correlation function for the $m = 1/3$ state for $\alpha = 0$ and $\alpha = 3.2$. The $\alpha = 3.2$ state shows incipient crystalline behaviour at short distances and liquid-like behaviour at long distances.

We may construct similar wavefunctions for $m = 5$ and $m = 7$.

$$\Psi_{\alpha}(z_i) = \prod_{i < j}^N [(z_i - z_j)(z_i - z_j - \alpha l)(z_i - z_j + \alpha l)(z_i - z_j - \beta l)(z_i - z_j + \beta l)] \times \exp\left(-\sum_i |z_i|^2/4l^2\right) \quad (4)$$

is an appropriate wavefunction for $m = 5$ and

$$\Psi_{\alpha}(z_i) = \prod_{i < j}^N [(z_i - z_j)(z_i - z_j - \alpha l)(z_i - z_j + \alpha l)(z_i - z_j - \beta l)(z_i - z_j + \beta l) \times (z_i - z_j - \gamma l)(z_i - z_j + \gamma l)] \exp\left(-\sum_i |z_i|^2/4l^2\right) \quad (5)$$

is an appropriate wavefunction for $m = 7$. We have not yet investigated these wavefunctions for all values of β and γ .

Particularly interesting cases are $\beta = i\alpha$ for $m = 5$ and $\beta = \omega\alpha$, $\gamma = \omega^2\alpha$ for $m = 7$ if ω is chosen as $\exp(2\pi i/6)$. The polynomial parts of the wavefunctions for this parameter choice may also be written as $\Pi z(z^4 - \alpha^4)$ (for $m = 5$) and $\Pi z(z^6 - \alpha^6)$ (for $m = 7$), where $z = z_i - z_j$. The correlation functions have a fourfold rotation axis for the $m = 5$

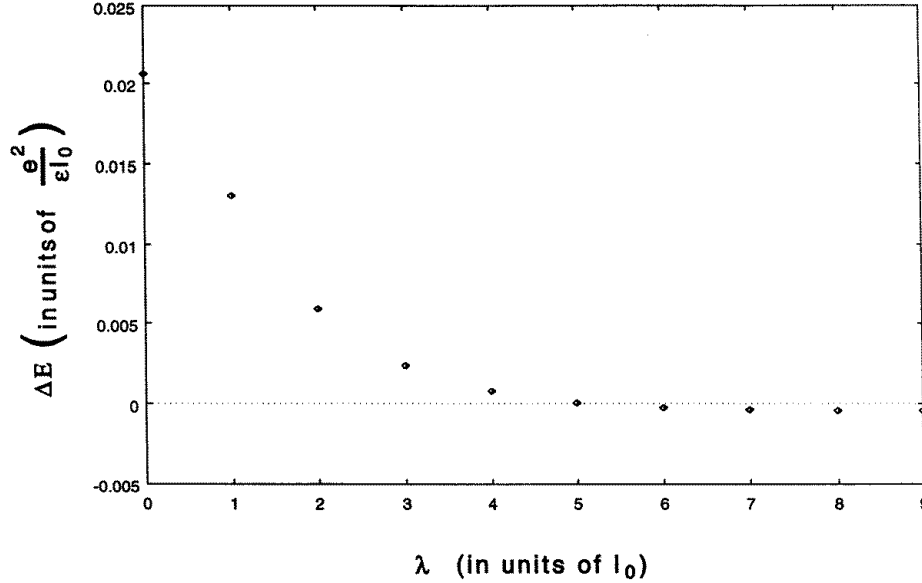


Figure 3. The difference between the energies per particle in a BRS state with $\alpha = 1$ and the Laughlin state as a function of λ , which is a measure of the well thickness.

wavefunction and a sixfold rotation axis for the $m = 7$ wavefunction. The incipient solid ordering again will stabilize these states as the well increases in thickness. For $m = 5$ and $m = 7$ we calculate critical values $\lambda_c(m = 5) = 2.9 \pm 0.3$ and $\lambda_c(m = 7) = 2.1 \pm 1.7$, respectively. The latter is a state which resembles the hexatic state of two-dimensional fluids.

The director $\mathbf{n} = (\text{Re}\alpha, \text{Im}\alpha)$ is the order parameter whose appearance signals the appearance of BRS. The states characterized by \mathbf{n} and $-\mathbf{n}$ are identical. There is no independent inversion symmetry operation in our two-dimensional system and thus the transition to the low-symmetry phase may be second order, unlike the situation for ordinary three-dimensional nematic systems. Our Monte Carlo calculations suggest that this is indeed the case, as the equilibrium value of α is a continuous function of t .

The Ginzburg–Landau free energy for an order parameter of this type is

$$F = A(T, t)n^2 + Bn^4 + K_1(\nabla \cdot \mathbf{n})^2 + K_2(\nabla \times \mathbf{n})^2 \quad (6)$$

where t is the thickness and T is the temperature. We expect a second-order transition when $T = 0$ and our calculations have been carried out only at zero temperature. At finite temperature thermal fluctuations are important.

The free energy expression shows that twists of the order parameter are possible and lead to textures but also to low-energy excited states. Nevertheless, these excited states do not involve density changes and the state as a whole is incompressible. We conclude that the FQHE still occur in this gapless state. The BRS state does not appear to be a candidate for the Hall insulator phase. The quasiparticle and quasihole excitations are still gapped and their charges are the usual fractional ones. Their charge density profiles will have elliptical distortion. Since the projected oscillator strength $f(\mathbf{k})$ and the projected static structure factor $S(\mathbf{k})$ depend on the direction of \mathbf{k} , the magnetoroton excitations with energy $E(\mathbf{k}) = f(\mathbf{k})/S(\mathbf{k})$ have dispersion which depends on the direction.

From the experimental point of view, it appears that the chief difficulty in identifying the BRS states lies in distinguishing them from the Laughlin state. Correlation functions are anisotropic, but scattering experiments to test this are difficult to perform in two-dimensional systems. Tensor quantities such as the conductivity have a characteristic anisotropy (birefringence): $\sigma_{xx}(\omega) \neq \sigma_{yy}(\omega)$, except at zero frequency, when $\sigma_{xx}(\omega = 0) = \sigma_{yy}(\omega = 0) = 0$, as usual. Propagation of surface acoustic waves or measurements of the microwave conductivity, perhaps with the simultaneous application of a current to eliminate domain effects, may be tools which can probe such an anisotropy, and test for the existence of the BRS states.

From the theoretical point of view, the chief difficulty is obtaining comparisons of BRS state energies and solid energies of the necessary accuracy. This is also a problem in the theory of hexatic phases in general [13]. We anticipate that further work will clarify this issue.

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